

# PROBABILITY PRACTICAL 1

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- (1) Blood groups are distributed in the UK as follows:

O	A	B	AB
48%	39%	10%	3%

- (a) If two people are picked at random from the population, what is the chance that their blood is of the same type? Of different types?
- (b) If four people are picked at random, let  $p_k$  be the probability that there are exactly  $k$  different blood types among them. Find all values of  $p_k$ .
- (2) In a genetic experiment, the offspring of a particular cross have 25% chance of being yellow, and 75% chance of being green.
- (a) If there are 10 offspring, calculate the probability that at least 8 are green.
- (b) Suppose they also have a 25% chance of being short and 75% chance of being tall. Calculate the expected number that are tall and yellow. What assumptions do you need to make?
- (3) Continuing an example from the lecture, suppose there is a disease that occurs in three forms: Mild, Severe, and Lethal. There is a gene that is known to occur in two *alleles* (variants), denoted  $A_1$  and  $A_2$ , where the  $A_1$  allele provides some protection against the symptoms of the disease, but does not prevent the disease. 75% of the general population has  $A_1$ , and 75% of those with the disease has  $A_1$ . Of those people with  $A_1$  who have the disease, 90% have the Mild form, and the rest have the Severe form. The  $A_2$  sufferers are evenly split between the Severe and Lethal forms.
- Suppose you observe a patient with the Severe form. Calculate the probability that the patient is of type  $A_1$ . Do this with Bayes' Rule and with natural frequencies.
- (4) I roll a fair die, and then flip a number of fair coins equal to the number that came up on the die.
- (a) Calculate the probability that exactly four heads come up on the coins.
- (b) Given that four heads come up, calculate the probability that the die roll was 5.
- (5) Suppose we have a sequence of independent trials, each with probability  $p$  of success. Let  $X$  be the number of the trial on which you have the first success.
- (a) Calculate the probability mass function  $\mathbb{P}\{X = k\}$ .
- (b) Calculate the expectation and variance of  $X$ .