

PROBABILITY PRACTICAL

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- (1) Suppose we have a sequence of independent trials, each with probability p of success. Let X be the number of the trial on which you have the first success.
 - (a) Calculate the probability mass function $\mathbb{P}\{X = k\}$.
 - (b) Calculate the expectation and variance of X .
 - (c) Toss a coin repeatedly, until you get heads. Record the number of flips. Then repeat, until you have at least 30 numbers.
 - (i) Estimate from your experiment $\mathbb{P}\{X \text{ is odd}\}$.
 - (ii) Calculate $\mathbb{P}\{X \text{ is odd}\}$.
- (2) Let X have uniform distribution on the interval $[a, b]$ defined as the continuous distribution whose density is constant on that interval and 0 outside it.
 - (a) What are the density and the cdf of this distribution?
 - (b) What are the expectation and variance?
- (3) Suppose there is a new infectious disease A. People can have the infection that leads to A for a long time before showing any obvious symptoms. About 1 person in 1,000 (0.1%) is currently infected.

Researchers have developed a blood test that is 99% accurate. That is, 99% of the time when the patient really is infected, the result is positive; and 99% of the time when the patient is not infected, the result is negative.

A doctor decides to perform this test on all of his patients. Suppose a random patient is tested and comes up positive. What is the probability that the patient is indeed infected?
- (4) In a certain country the heights of adult males have mean 170cm and standard deviation 10cm, and the heights of adult females have mean 160cm and standard deviation 8cm; for each sex the distribution of heights approximates closely to a normal probability model. On the hypothesis that height is not a factor in selecting a mate, calculate the probability that
 - (a) a husband and wife selected at random are both taller than 164cm
 - (b) in a randomly selected husband and wife the wife is taller than the husband
 - (c) the average height of a random couple is greater than 168cm. [You may use the fact that the sum of two independent normal random variables is also normal.]

Note: You may compute the cdf for the standard normal random variable in **R** with the command `pnorm(z)`.

- (5) In late 1996 a woman named Sally Clark suffered the apparent cot death of her first child. In early 1998, her second child also died suddenly and unexpectedly. Partly on the grounds that this was a highly unusual occurrence, and partly on the basis of expert evidence, she was charged with the murder of both children.

At her trial in 1999, the distinguished paediatrician Professor Sir Roy Meadows reported the results of an earlier study that estimated the probability of 1 cot death befalling a woman in Mrs Clarks position (conditional on several environmental variables, including the fact that she and her

husband were both non-smokers) as 1 in 8,540. Prof. Meadows went on to testify that the probability of 2 such cot deaths was around 1 in 73,000,000.

Sally Clark was subsequently convicted, with a newspaper report at the time stating that the expert evidence showed that “the chance of the mother being innocent is 1 in 73,000,000”.

- (a) How was the 1 in 73 million figure obtained? What assumption(s) does this involve? Comment on possible problems with this approach.
- (b) Suppose now that the 1 in 73,000,000 claim of Prof. Meadows is in fact correct. Is the newspaper report reasonable?

[Some background on the Clark case]

Sally Clarke’s case was the subject of an appeal, heard by the Court of Appeal in 2000.

One issue at Appeal was whether the trial judge had been right to allow the charge in respect of the death of each of Mrs Clarks children to be heard together, rather than as two separate murder trials. The legal issue here has to do with how similar the facts of each case are. (They need to be strikingly similar to allow both charges to be tried together, to override the general principle that one offence a person has committed should not be admitted as evidence at their trial on another matter.)

In summary of the evidence, the Court of Appeal wrote the following (Mr Bevan was the Barrister for Clark, Mr Spencer the Barrister for the Crown):

In expressing his conclusions in the present case, the judge held first that he should take into account all the circumstances, including the medical evidence, relating to the death of each child. On the other hand he accepted that it would be quite wrong to adopt the approach that if the defendant had killed one of the children, it made it more likely that she must have killed the other. He set out the correct legal test, drawn from DPP v. P. He bore in mind that the alleged cause of death in each case was different, as was the case at the time of his ruling, but took the view that that fact by itself did not affect the principle of admissibility. Thirteen similarities had been identified by the prosecution but he based his decision primarily on the first six of them. He said that his decision necessarily involved a value judgment of the degree of probative force of those matters. His decision was based on their cumulative force, although he did attach considerable importance to the sixth similarity. The first six similarities referred to by the judge were these: “(1) Both babies were, according to the defendant, found unconscious by her in the same room and apparently in the same bouncing chair, which the medical evidence suggested was not what one would expect in a SIDS’ (or cot death) case. Mr Bevan pointed out in respect of that that babies are generally looked after at home; but Mr Spencer replied that this is not just a matter of being in the same house. (2) Both babies were found by the defendant at almost exactly the same time of the evening, namely about 9.30 p.m. which, it was said, was inconsistent with a SIDS’ case. Mr Bevan asks rhetorically what difference it would have made if it was 4.30 p.m. Mr Spencer pointed out that in both cases the babies had taken a feed shortly before their death. (3) Both babies died at about the same age: Christopher 11 weeks,

Harry 8 weeks. Mr Bevan submitted that the evidence was that cot deaths are most common with young babies of up to three months, but Mr Spencer pointed out that the coincidence was still there nevertheless. (4) In each case the defendant was alone [with] the baby at the time of discovery. Mr Bevan submitted that that was of no significance at all, whereas Mr Spencer said that it was significant because in the case of Harry on the evidence the defendant had only been [alone] with the baby a short time of four minutes or so. (5) In each case the defendant's husband was either away or about to go away from home in connection with work. Mr Spencer referred to evidence showing that she was resentful of being left on her own and tended to drink more heavily when her husband was away. In the case of Christopher he was away at an office party. In the case of Harry, he was about to go to Glasgow on business the next day. In the case of Harry the defendant visited the off-licence on two occasions to buy some wine, saying (falsely, it would appear) that they were having a dinner party that evening. Mr Bevan accepted the factual circumstances, but submitted that there was no significance in those circumstances, whereas Mr Spencer submitted that it was significant because it was an unusual feature that he was either away or about to go away on both occasions. (6) In each case there is evidence of previous abuse prior to the fatal episode: in Christopher's case, asphyxiation; in Harry's case, shaking. Mr Spencer said that if both deaths were natural deaths, what a coincidence it would be if, in each case, nevertheless there was evidence of unexplained previous abuse. In the case of Christopher, there was a great deal of old blood in the lungs, which was unexplained and was consistent with smothering; and in the case of Harry, there was evidence of shaking on at least one previous occasion. In relation to that, Mr Bevan pointed to the evidence of Dr Keeling that, even in SIDS' cases, one can find recent haemorrhage, and, although this related to old haemorrhage, that logic should dictate the same approach. Mr Spencer referred to Dr Keeling's evidence that in relation to both the old and the new haemorrhage, in her view there was no natural explanation for it."

How much weight would you put on these similarities?

At the Appeal in 2000, Sally Clarks conviction was upheld.

There was a later Appeal in 2003 - her convictions were quashed. One central feature of the second appeal was that key medical evidence showing possible signs of natural illness of the Clarks second child at the time of his death had been hidden by one of the prosecution medical experts.

Interestingly, at the first appeal, the Court of Appeal argued that Meadows statistical evidence would not have been likely to have made a great impact on the jury. At the second appeal, the Court of Appeal was very critical of him. Many other cases involving his evidence have subsequently been reinvestigated, and several women subsequently freed. On 15 July 2005 Meadows was struck off the medical register for giving erroneous and misleading evidence which helped to convict Sally Clark of murdering her two sons. (The Times, 16 July 2005)